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Identification of the significant uncertain parameters in the seismic response of irregular bridges

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ABSTRACT

Developing probabilistic seismic demand models is a key element in seismic risk assessment of structures such as bridges. Identifying the influential parameters related to the seismic response of a structure is a crucial step towards evaluating its seismic vulnerability. Most of the past related sensitivity studies have focused on regular bridges with typical configurations, although observed damage from past earthquakes affirms that compared to regular bridges, those with irregularities or geometric inconsistencies in the configuration are more susceptible to noticeable damage. In this paper, using state of the art statistical methodology, the influence of various parameters on the resulting probabilistic seismic demand is investigated. This study concentrates on concrete bridges including three geometric irregularity types: (i) skew angle, (ii) a frame with unbalanced stiffness, and (iii) tall column heights, and a comprehensive sensitivity of a broad range of probabilistic modeling parameters on the seismic response is assessed. The statistical analysis reveals that the common parameters including ground motion intensity, longitudinal reinforcement ratio, column diameter, number of columns per bent, column height, span length, and concrete compressive strength significantly influence the response of the three studied irregular bridges. The individual influential parameters affecting each class of irregularity are highlighted and discussed.

1. Introduction

The experience of past earthquakes (Northridge in 1994, Kobe in 1995, and Chile in 2010) reveals that compared to bridges consisting of typical configurations, bridges with irregularities in their configuration have a higher chance of being severely damaged or collapsed [1–3]. Bridges with typical geometric configurations are defined as regular bridges (i.e. bridges with zero skew angle, zero curvature, normal column heights, and balanced stiffness between frames). The primary objective of this paper is to improve the understanding of the seismic performance of irregular bridges through statistical sensitivity studies, while most of the past studies focused on regular bridges.

Probabilistic seismic demand models (PSDMs) are essential tools to describe the seismic demand of various components of a bridge in terms of the ground motion intensity measures. Researchers [4–8] commonly utilize PSDMs to perform fragility analysis for characterizing the conditional reliability of bridges. Several studies [4,9,10] conducted sensitivity studies to identify the parameters significantly affecting the bridge response. Nielson and DesRoches [4] and Padgett and DesRoches [10] studied one specific bridge type, three span simply supported steel girder bridge that was a non-skewed bridge with normal column heights and balanced frame in the central US. Nielson and DesRoches [4] considered 14 input variables including concrete and steel strength, coefficient of friction for bearing, initial stiffness of bearing, initial stiffness of passive and active abutment, rotational and translational stiffness of foundation, mass, damping ratio, gap between abutment and deck, gap between decks, and ground motion loading direction. Later, Padgett and DesRoches [10] addressed a research gap for the retrofitted bridges by adding retrofit parameters for the restrainer cable, elastomeric bearing, steel jackets, and shear key, to the sensitivity study and conducting fragility analysis sensitivity on this bridge type. Duke, et al. [9] illustrated a sensitivity analysis on a two span integral concrete box girder bridge in California and investigated the effect of five design parameters listed as longitudinal and transverse reinforcement ratio, column height to the column dimension ratio, superstructure depth to the column dimension ratio, and span length to the column height ratio. These previous studies typically focus on regular bridges and selected numerical parameters. However, the present

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study attempts to determine the influence of a broader range (43 input variables) of modeling parameters including categorical parameters as well as numerical parameters on the seismic response of irregular bridges.

Among geometric irregularities, skew requires particular attention mainly because past earthquakes have shown serious damage caused by the displacement or the unseating of the bridge decks in bridges with large skew angles [11]. Skewed bridges are commonly constructed in response to complex geometric constraints at the site that necessitate using skew-angled abutments. As a result, the eccentric passive resistance of the abutment backfill initiates and promotes the in-plane rotation of the superstructure's deck that may ultimately cause unseating of the superstructure leading the skewed bridge to collapse. Several research efforts investigated the effect of skew angle along with a few other parameters on the seismic response of bridges [12–15]. As an example, Abdel-Mohti & Peckan [12] explored the seismic performance of a selected three-span concrete box-girder bridge with three different skew angles of 30°, 45°, and 60°. They made comparisons for bridges with two different cases of boundary conditions (with and without shear keys) and directions of applied ground motions. Four selected ground motions were applied for pushover and time history analysis using SAP2000. The results showed skew angle and boundary conditions to have significant effects on the bridge response. Meng & Lui [13] analyzed a 60° skewed box-girder bridge, the Foothill Boulevard Undercrossing, in SAP2000. The effect of deck flexibility, column base fixity, and skew angle was investigated and it was concluded that all three factors can change the bridge response drastically.

Kaviani et al. [11] performed a seismic assessment of selected skewed bridges located in California. Three short California bridges with various structural parameters have been selected. Sensitivity of the deck rotation and column drift ratio was assessed according to five skew angles 0°, 15°, 30°, 45°, and 60°, two different span ratios 1.0 and 1.2, two column height ratios 1.0 and 1.5, and three types of ground motions as soil-site, rock-site, and pulse-like with six angles of incidence, 0°, 30°, 60°, 90°, 120°, and 150°. It was demonstrated that the monitored skewed bridge demands including deck rotation and column drift ratio were higher than those for the non-skewed bridges, and these demands exhibited sensitivity to the characteristics of the ground motions as well as the skew angles. Among other studies in this area, Sullivan [16] and Yang et al. [17] developed fragility curves of bridges with skew angles between 0° to 45° located in the moderate seismic zones. Sullivan [16] conducted fragility analysis and developed associated curves for skewed and multi-span simply-supported steel girder bridges. The proposed fragility curves indicate that the bridge fragility is not noticeably influenced by the low to medium skew angles (i.e., skew angles less than 30°), while it is significantly affected by the higher range of skew angles. Yang et al. [17] evaluated bridges with various design types, and retrofitting strategies. They found that overall, bridges with larger skew angles are more vulnerable to seismic excitations. Zakeri et al. [18] investigated the effect of skew angles, single and two-column bents, integral and seat-abutment types, and seismic design levels on the fragility curves of concrete box-girder bridges. The old designed bridges showed less sensitivity to the skew angle compared to the more recently designed bridges, and bridges with integral abutment types exhibited less vulnerability to the skew angle in comparison with bridges with seat-type abutments. Similar to the past sensitivity studies on regular bridges, the mentioned studies on skewed bridges investigated the influence of selected parameters such as skew angle, direction of ground motion, and span ratios on several bridge responses such as deck rotation and column drift ratio. In order to provide a more comprehensive perspective on the impact of modeling parameters on the seismic response of skewed bridges as a type of irregular bridge, this paper conducts sensitivity study on a broader range (43 variables) of modeling input parameters on various component responses (9 component responses) which are commonly used to develop PSDMs and system fragility curves. The previous studies have been invaluable in enhancing knowledge about the response of bridges. However, none of the cited previous studies used the recently improved methodologies to create numerical modeling of skewed abutments, a broad range of input variables for the sensitivity study of concrete box girder bridges, and recently developed and more robust statistical techniques for the sensitivity study. Therefore, there is still a need to a better understanding of issues such as the most significant modeling parameters on the responses of various bridge components and a quantitative measure of the relative importance of these parameters.

Other types of irregularities in bridge configuration include a frame with unbalanced stiffness and tall column bents. Bridges with these irregularities are typically constructed in specific regions (e.g., mountainous areas, deep valleys, and overcrossings) with complex topography for the foundation layout. Consequently, based on the topography attributes, some of these bridges have columns higher than the typical range, while others have columns of variable height. According to the post-earthquake observations, Zheng & Wenhua [19] explored the main four failure modes of bridges with high or non-uniform columns in mountainous areas. Based on their study, the first damage state was associated with a change in the position of the abutment, abutment settlement, and damage to the superstructure deck. The second state was mainly related to the cracking and breaking of piers, in addition to the buckling of the steel reinforcement. The third state of damage was caused by the inclination and deterioration of supports and the last state of damage resulted in the bridge collapse because of the failure of piers and supports followed by the falling of the superstructure. In the case of a bridge frame with unbalanced stiffness, the large relative displacement between the adjacent piers [20] with inconsistent column heights is the major factor affecting the superstructure's failure. The combination of tall and short piers within a bridge exposed to earthquake excitation results in uneven force distribution between the piers [20,21]. Zheng & Wenhua [19] also clarified the importance of following a separate seismic design procedure for tall-pier bridges. They recommended using stronger column bents to be able to resist large bending moments, shear forces, and torques. All of these factors indicate the complex seismic response of bridges with unconventional column attributes. Jara et al. [22] examined the effect of three different topologies of unequal column heights on the seismic demand of the bridge columns. The selected medium length bridges included two cases of five-span bridges and one case of a six-span bridge. Twelve ground motions and two soil types, soft and hard, were selected for the analysis. Both the considered unequal configurations and soil types showed significant impact on the pier damage index, particularly for the columns located adjacent to the tallest column. Abbasi et al. [23] analyzed the seismic fragility of old designed box-girder viaduct bridges with an expansion joint and four levels of variations between the column heights. The studied bridge was a four-span bridge with three columns per bent. The results demonstrated that the fragility of the considered bridges increases by increasing the variation between the column heights, and among various components of the bridge, deformation of the bridge deck and the in-span hinge presented the highest sensitivity to the height variation. That study is limited to one specific bridge type and only considered the effect of column heights on the bridge response. The seismic response and performance of tall and unbalanced bridges have not been deeply studied, and hence there is a need to further assess the seismic performance of these irregular bridge configurations. To date, sensitivity studies on the impact of modeling parameters on the seismic response of unbalanced and tall bridges have not been performed and therefore this paper aims to address this deficiency.

One of the leading steps toward developing a more reliable and realistic seismic fragility framework of irregular bridges is the completion of systematic sensitivity analysis to identify the influential uncertain parameters related to the key responses. Moreover, this paper aims to determine characteristics corresponding to the general configuration of irregular bridges that have the most significant effect on the bridge response. Regression analysis incorporated with hypothesis testing is a popular approach that helps identifying the impact of parameters involved in the response. Although the selected parameters in the past studies were commonly selective and quantitative variables, this study evaluates the effect of a more comprehensive list of modeling parameters including both quantitative and qualitative ones using statistical tools including Categorical Regression Analysis and Lasso Regression, and Partial F-statistics. This study addresses a wide range of irregularities according to the existing California bridge inventory. The considered ranges include skew angles varying from zero to 77°, tall column height ratios (i.e. ratio of the average column height of a tall bridge to the average column height of a normal bridge) ranging from 1.5 to 4.5, and an unbalanced frame with stiffness ratios (i.e. stiffness ratio between different bents within a bridge frame) changing from 75% to lower than 15%. The details of the irregularity ranges are provided in Section 2.

The next section (Section 2) reviews the general procedure for generating three-dimensional numerical bridge models in OpenSees. An accessible inventory of existing irregular bridges located in California serves as the basis for establishing bridge component characteristics. This study considers a complete list of uncertain characteristics as input parameters for statistical analysis. Critical seismic responses serve as outputs of the statistical model and are captured by performing nonlinear time history analysis.

The remainder of the paper is arranged as follows. In Section 3, implemented statistical approaches are described to deal with both numerical and categorical variables. Following that, Section 4 presents the implementation process and comparisons of the results in three main aspects. First, the effects of parameters are discussed separately for various ranges within each irregularity type. Second, comparisons are made between irregularity types and their associated significant parameters. Third, weights of significant parameters are examined to measure their effectiveness in predicting the bridge responses. The paper concludes by detecting common influential parameters, although the relative significance of the various predictors changes over different bridge responses and irregularity ranges.

2. Numerical modeling and analysis

2.1. Numerical modeling of bridges

For the purpose of this study, the single-frame box-girder concrete bridge type is selected, and OpenSees was used to develop three-dimensional numerical models. Fig. 1 illustrates the general layout and approach for numerical modeling of the bridges. The incorporation of skew into the analytical modeling of straight bridges necessitates various modifications including recently developed modeling strategies based on the experimental and numerical studies of skewed bridges [24,11]. In this study, skew angles are divided into 5 ranges: low (0°–15°), medium (15°–30°), high (30°–45°), very high (45°–60°), extreme (60°–77°), noted as the maximum value in the database for the existing bridges in California. In the simulation process, skew angles are distributed uniformly in each range and assigned randomly to the bridge samples.

To date, limited research exists regarding bridges with tall column bents and a frame with unbalanced stiffness. Likewise, to the best of the authors' knowledge, there are no data sets from experiments on these classes of bridges. Therefore, the only change that is considered in the present study for modeling bridges with unbalanced frames and tall column bents is the variation of the column heights. In this regard, four groups of California box-girder bridge plans, listed as stream crossings, ramps, connectors, and viaducts were reviewed in detail to extract column height values in order to have a comprehensive database. The overall goal of this step was to set up a realistic sample of unbalanced and tall bridge profiles to use for creating synthetic unbalanced and tall bridge realizations in the analytical modeling of bridges. The average column heights (H_{ave}) of unbalanced and tall bridges were normalized by the average column heights (H_{base}) of the base models (i.e., regular bridge with normal column height and balanced stiffness frame). Those ratios of H_{ave}/H_{base} meeting the column heights criteria to be considered as tall bridges (i.e., ratios higher than 1.5) are used for building the models for this class of bridges. As an example, the ratios for bridges designed in the Pre-1971 era are shown in Fig. 2a. The column height ratio of tall bridges is divided into three different ranges as shown in Fig. 2b including: moderately tall ($1.5 \leqslant H_{ave}/H_{base} < 2.5$), very tall ($2.5 \leqslant H_{ave}/H_{base} < 3.5$), and extremely tall ($3.5 \leqslant H_{ave}/H_{base} < 4.5$).

A single column-height-ratio is developed by normalizing the height of all the columns in a bridge (H_i; $i = 1, \ldots, n$ and n = number of columns in a bridge) to the bridge-average-height (H_{ave}). This yields ratios (H_i/H_{ave}) for each bridge that is centered on 1, but values extend both above and below the center (Fig. 3a). Based on the Caltrans Seismic Design Criteria [25], a bridge is defined to have a frame with unbalanced stiffness when different bents within the frame have a stiffness ratio of less than 75%. Since stiffness is a function of modulus of elasticity, moment of inertia, and column height, the criteria assigned to the stiffness ratio of an unbalanced frame can be converted to a criterion for the column heights, by assuming similar modulus of elasticity and moment of inertia for different bents in a frame. This criterion is converted to column height ratios by normalizing the short (H_1) and tall (H_2) column heights of a bridge by the average column height, as shown in Fig. 3b. The ratios of H_1/H_{ave} and H_2/H_{ave} are calculated as 0.95 and 1.05, respectively. Thus, the respective ratios (i.e., higher than 1.05 and lower than 0.95, Fig. 3a) are implemented in the modeling of bridges with unbalanced frames. For bridges with unbalanced stiffness, four ranges of column height ratios are considered as (Fig. 3b): slightly unbalanced (55% ⩽ stiffness ratio < 75%), moderately unbalanced (35% ⩽ stiffness ratio < 55%), highly unbalanced (15% ⩽ stiffness ratio < 35%), extremely unbalanced (stiffness ratio < 15%). The corresponding column height ratios for the short column H_1/H_{ave} and for the tall column H_2/H_{ave} are provided in Fig. 3b.

Table 1 summarizes the various levels of irregularities discussed above. The three-dimensional bridge models are created in OpenSees for each level of irregularity, two different types of abutments (i.e. rigid diaphragm and seat-abutment types), and the specifications of various design eras (i.e. bridges designed before 1971, between 1971 and 1990, and after 1990). The bridge deck elements are typically modeled using elastic beam column elements as the bridge deck remains elastic during earthquake. The bridge columns are modeled using displacement beam column elements with fiber cross sections [26]. The column elements are connected to the deck elements and bridge footing by rigid links and foundation springs, respectively. More details (e.g. modeling

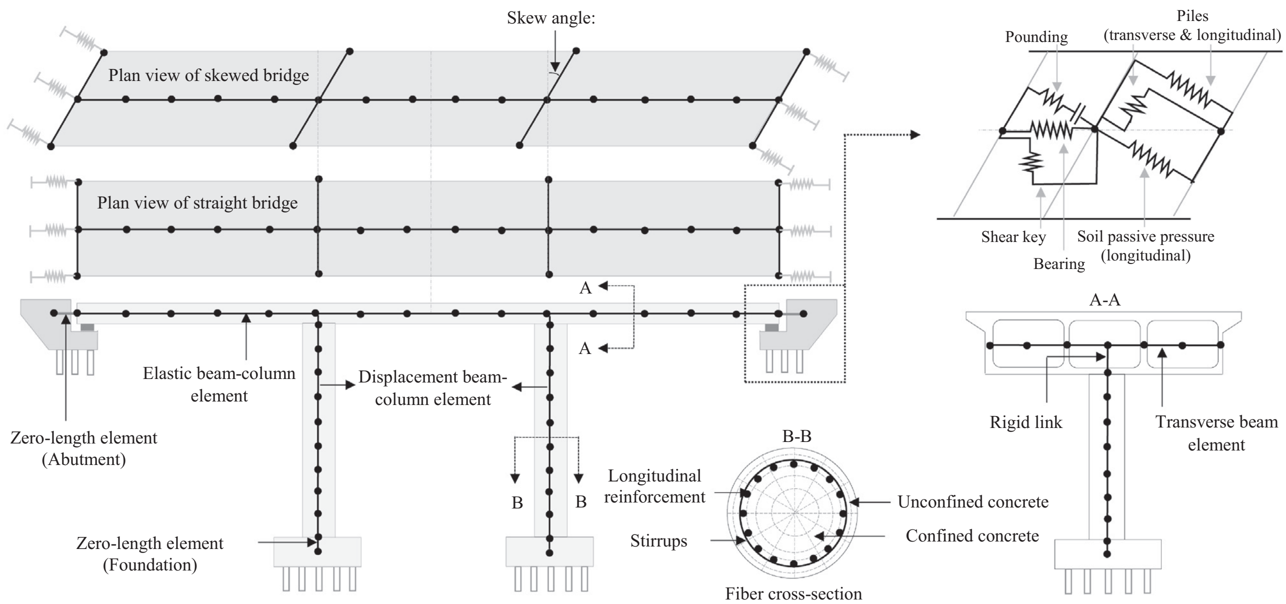

Fig. 1. Typical layout of a single-frame, three-span, box-girder bridge.

of pounding and bearing elements) about numerical modeling of box-girder bridges can be found in the previous studies [4,7].

2.2. Statistical parameters

The statistical parameters in this study can be classified into two main groups of categorical and numerical parameters. In this study, $S_{a\text{-}1.0s}$ (i.e. the spectral acceleration at 1.0 s) was assigned to the first potential predictor X_1 and chosen as the measure of the ground motion intensity, since it has been found to be the optimal intensity measure for classes of box-girder bridges [7]. The associated parameters of the two groups are listed in Table 2. To incorporate the uncertainty into the numerical modeling of bridges, probabilistic geometric and material parameters were selected for the numerical simulations (Table 3). The column height values in addition to the distribution parameters are listed in Table 4. All column height values follow the lognormal distribution.

3. Statistical analysis framework

In order to detect the influential parameters on the seismic responses of the irregular bridges, this study deals with a problem that involves both numerical and categorical variables. In the following, categorical regression analysis, applicable to the models with only categorical variables, is explained through an illustrative example. Then, a detailed explanation is provided for the Lasso Regression that treats categorical variables similar to the process used in the categorical regression analysis. Later, Lasso Regression is implemented to identify the parameter weights in predicting the bridge response.

3.1. Categorical regression analysis

The majority of regression models focus on numerically valued variables (i.e. variables that are measured in a numerical scale). However, in this study, some of the parameters considered are qualitative (ordinal and categorical) (Table 2). Contrary to the numerical variables, the effect of categorical variables cannot be estimated using standard regression models. Thus, a categorical regression method is applied herein to incorporate the qualitative variables into the regression model.

The categorical regression method introduces a set of *indicator* or *dummy* variables to account for the different levels of a variable and even more importantly to obtain variables in the regression model that have simple interpretations. As an example, to introduce the effect of two separate levels of a variable (e.g. abutment type) into a binary regression model, an indicator variable is defined as:

$$x = \begin{cases} 0 & \text{if the variable is in category } \#1 \\ 1 & \text{if the variable is in category } \#2 \end{cases} \tag{1}$$

In some applications, the variable is not binary, but rather multi-categorical (e.g. design era). When categorical variables with more than two levels are included in a regression model, additional steps are required to ensure the consistency and interpretability of the results. These steps consist of recoding categorical variables into a number of separate variables. Hence, in a general case, a variable with k possible levels of category is modeled by $k-1$ indicator variables. For example, if a categorical variable X has five levels, then it will be transformed into four separate variables $(x_1, \ldots, x_4)$ that will be used in the multiple regression model and contain the same information as the initial single categorical variable. The indicator variables are assigned either zero or unit values, representing each level of the category. Thus, the binary coding takes the following form:

$$x_i = \begin{cases} 1 & \text{if the variable is in category } i \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

where $i = 1, \ldots, k-1$, and the k^{th} category is selected as the reference. In this approach, indicator variables can be included in the hypothesis testing similar to any other variable. Their mean differences can be estimated with a linear model by representing groups with a set of $k-1$ variables, where k is the total number of groups. An alternative coding approach is using -1 and 1 instead of 0 and 1. The only difference between these two coding strategies is raised in the interpretation.

According to this standard, when the response y depends solely to one categorical variable, X, the predictor is modeled by multiple dummy variables as

$$E(y) = \beta_0 + \beta_1 x_1 + \ldots + \beta_{k-1} x_{k-1} \tag{3}$$

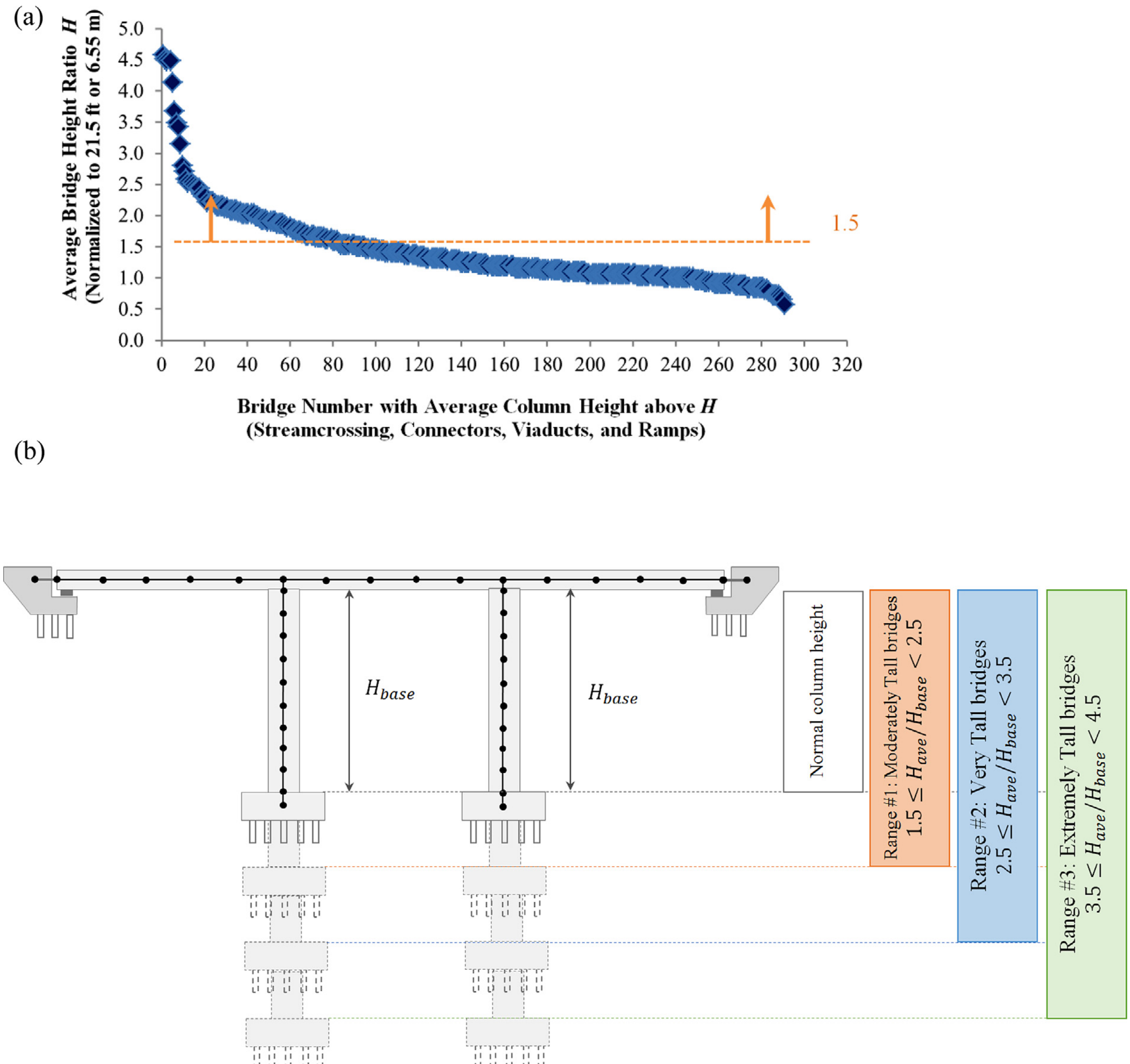

Fig. 2. (a) Average bridge column height ratios for bridges designed in Pre-1971; (b) Considered configurations for tall bridges.

Interpretation of this categorical model follows directly from determining the response for different categories of X. If the model is in the k^{th} category, $E(y) = \beta_0$ which means the intercept in the categorical regression model represents the expected response for the reference category (i.e. k^{th} level). If the model is in the i^{th} category, then $E(y) = \beta_0 + \beta_i$ which means each slope β_i in the model indicates the increase or decrease of the expected response in comparison to the reference category k. Hence, the results can be interpreted as the change in the expected transition from one category to another.

The categorical regression analysis yields a model mathematically identical to Analysis of Variance (ANOVA) with similar interpretations and statistical inferences. Consequently, the categorical regression weights are equal to the ANOVA mean differences. Both of these techniques retain the information on how the k groups differ from one another, which determine the influence of the parameters considered.

3.2. Lasso regression

Conventional regression techniques mainly follow the standard least squares framework by considering all possible covariates in the model in spite of the fact that the resulted estimates are not often satisfactory. The first concern is associated with the prediction accuracy, as the estimates generated by the least squares method can have large variances. The second challenge relates to the interpretation of the developed regression model, which is intricate in the case of a model with a large number of predictors.

In order to cope with these problems, a number of approaches are proposed and known as the variable subset selection techniques [28,29], such as *Best-Subset Selection* [30], *Forward-Stagewise Regression* [31], *and Forward-stepwise and backward-stepwise Selection* [32], which improve the prediction accuracy. Generally, the subset selection reduces the variance of the estimates and prunes some of the predictors with less impact on the overall regression model. Although these techniques produce improved models, they utilize a discrete process in which a variable is either retained or discarded. As a result, these techniques may perform poorly in reducing the prediction error of the full regression model.

Shrinkage methods [33] are more recently developed tools that use a continuous process rather than the discrete scheme, which lead to noticeably reducing the variance and the prediction error. These methods minimize the residual sum of squares subject to a

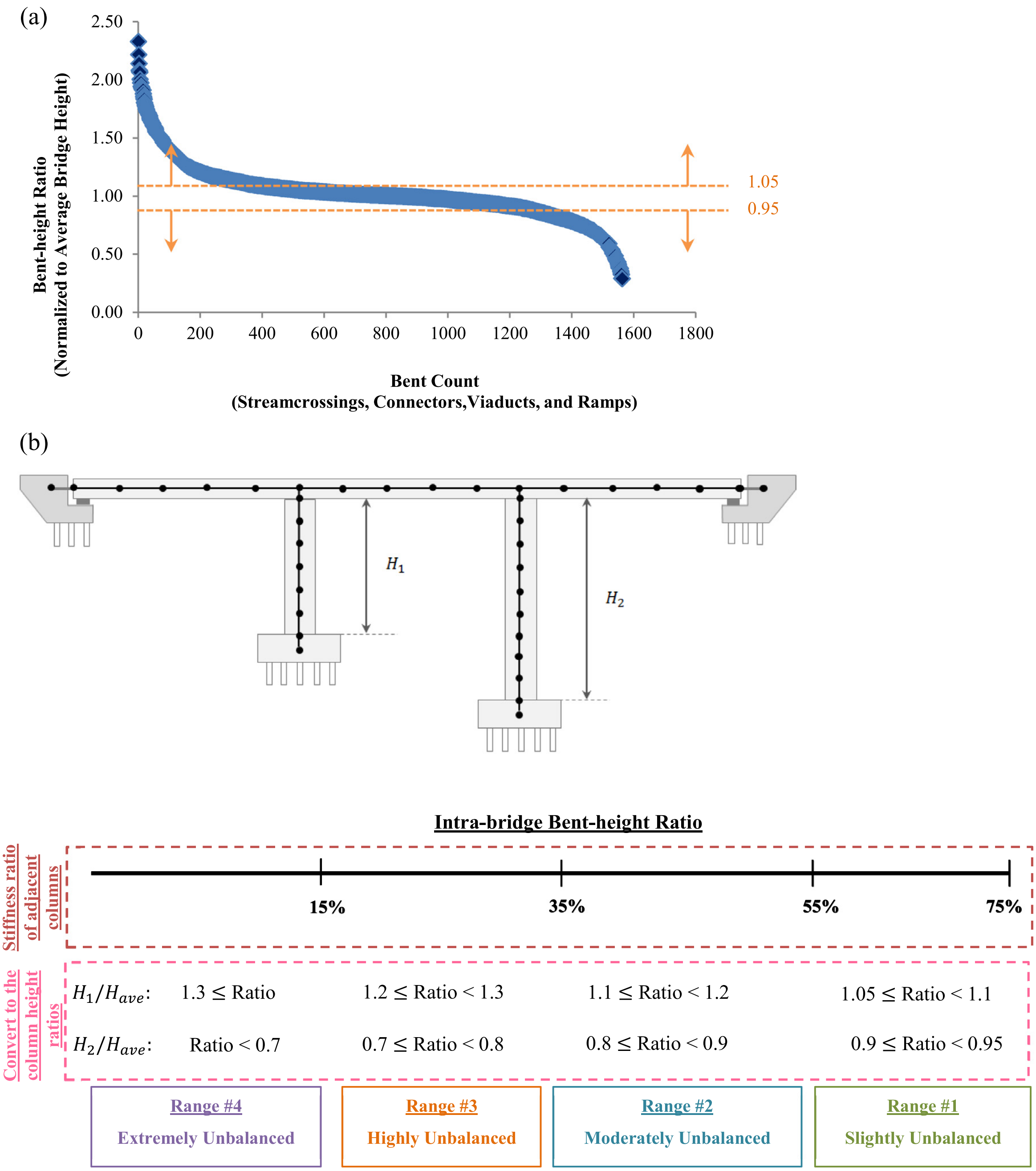

Fig. 3. (a) Bent height ratios for bridges designed in Pre-1971; (b) Considered configurations for a bridge frame with unbalanced stiffness.

Table 1
Summary of the considered ranges for irregularity parameters.

Irregular Parameters	Levels
Skew	Low, Medium, High, Very high, Extreme
Tall	Moderately Tall, Very tall, Extremely tall
Unbalanced	Slight, Moderate, High, Extreme

constraint on the magnitude or the cardinality of the coefficients, which is their main distinction from the previously mentioned approaches. Such restriction controls the model complexity, which subsequently controls the variance of the predicted values and improves the overall prediction accuracy. Ridge Regression and Lasso Regression are among the most well-known shrinkage techniques, and between these two, Lasso overcomes Ridge in several aspects that are explained further. A more comprehensive study and comparison of these methods are presented by James et al. [34].

Lasso (Least Absolute Shrinkage and Selection Operator) is a robust statistical regression technique, mainly applicable to the problems with a large number of covariates from which the influential set needs to be determined. Consider y_i as the i^{th} response that depends upon the variables x_{ij}, $i = 1, \ldots, n$ and $j = 1, \ldots, p$, where n and p denote the number of collected data for the response and the number of regressors, respectively. Lasso estimates the coefficients of the regression model through the convex constrained minimization

Table 2
Description of the potential predictors for the statistical analysis.

	Parameter	Categories	Assigned Variable
Categorical Input Parameters	Abutment type	Rigid diaphragm, seat	X_2
	Design era	Pre-1971, 1971-1990, Post-1990	X_3
	Number of columns per bent	Single, Two, Three, Four, Five	X_4
	Soil type	Clay, Sand	X_5
	Superstructure box type	Reinforced concrete, Pre-stressed concrete	X_6
	Number of box cells	Three, Five, Seven, Nine, Eleven, Fifteen	X_7
	Direction of applied excitation	Longitudinal, Transverse	X_8

	Parameter	Unit	Assigned Variable	Parameter	Unit	Assigned Variable
Numerical Input Parameters	Ground motion intensity measure	(g)	X_1	Restrainer stiffness	(kN/cm)	X_{26}
	Span length	(m)	X_9	Restrainer yield deformation	(mm)	X_{27}
	Column height	(m)	X_{10}	Number of restrainers	N/A	X_{28}
	Deck width	(m)	X_{11}	Concrete compressive strength	(Mpa)	X_{29}
	Girder spacing	(cm)	X_{12}	Reinforcing steel yield strength	(Mpa)	X_{30}
	Top flange thickness	(cm)	X_{13}	Shear key capacity	(kN)	X_{31}
	Bottom flange thickness	(cm)	X_{14}	Multiplicative factor for coefficient of friction of bearing pads	N/A	X_{32}
	Wall thickness	(cm)	X_{15}	Shear modulus of elastomeric bearing pads	(Mpa)	X_{33}
	Depth of superstructure	(cm)	X_{16}	Transverse gap between deck and shear keys	(mm)	X_{34}
	Column diameter	(cm)	X_{17}	Longitudinal gap between deck and abutment	(mm)	X_{35}
	Longitudinal reinforcement ratio	N/A	X_{18}	Pile stiffness	(kN/cm)	X_{36}
	Confinement spacing	(cm)	X_{19}	Mass factor	N/A	X_{37}
	Abutment backwall height	(m)	X_{20}	Damping	%	X_{38}
	Pile spacing	(m)	X_{21}	Ground motion time step	(sec)	X_{39}
	Foundation translational stiffness	(kN/cm)	X_{22}	Skew angle	(degree/ °)	X_{40}
	Foundation rotational stiffness	(kN-m/rad)	X_{23}	Tall ratio	N/A	X_{41}
	Restrainer length	(m)	X_{24}	H1_ratio	N/A	X_{42}
	Initial slack in restrainer cable	(mm)	X_{25}	H2_ratio	N/A	X_{43}

$$\hat{\beta}^{Lasso} = \underset{\beta}{\text{argmin}} \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j \right)^2, \quad \text{Subject To}$$

$$: \sum_{j=1}^{p} |\beta_j| \leqslant t(\text{constant}), \tag{4}$$

which minimizes the residual sum of squares, subject to an ℓ_1-norm constraint on the coefficients. Such constraint is shown to promote sparsity among β coefficients.

This problem has an equivalent matrix form

$$\hat{\beta}^{Lasso} = \underset{\beta}{\text{argmin}} \left\{ \underbrace{\| \mathbf{Y} - \mathbf{X}\boldsymbol{\beta}_2^2 \|}_{\text{Lossterm}} + \underbrace{\lambda \|\boldsymbol{\beta}\|_1}_{\text{Penaltyfunction}} \right\}, \tag{5}$$

where $\boldsymbol{X}$, $\mathbf{Y}$, and $\boldsymbol{\beta}$ are the matrix form of regressors x_{ij}, vector of responses y_i, and vector of regression coefficients, respectively. The tuning parameter λ is directly related to the constant t and controls the generated models, such that when λ is sufficiently large, t is equivalently small.

The advantage of Lasso over Ridge relates to the penalty function which is the source of difference between their performances. In this regard, Lasso applies an ℓ_1-norm penalty on the coefficient vector $\boldsymbol{\beta}$ (i.e. $||\boldsymbol{\beta}||_1 = \sum_{j=1}^{p} |\beta_j|$), while Ridge imposes an ℓ_2-norm constraint on $\boldsymbol{\beta}$ (i.e.$||\boldsymbol{\beta}||_2 = \sum_{j=1}^{p} \beta_j^2$). Thanks to the geometric structure of the ℓ_1-ball, the coefficient estimates of the parameters x_{ij} with the least impact on the response y_i are forced to be exactly zero.

Fig. 4 shows the geometrical illustration of Eq. (5) and the interpretation of the Lasso and Ridge constraints for a model with two parameters. Technically speaking, the first point of contact between a sub-level set of the loss and the penalty ball characterizes the optimal solution. This initial contact point can be located at a corner of the Lasso diamond-shaped region and thus one of the two coefficients vanishes to zero. As the number of parameters

Table 3
Distribution of modeling parameters (source: review of bridge plans [27,7].

Parameter	Unit	Distribution	Distribution parameters	
			Factor 1*	Factor 2**
Span length	(m)	Empirical	35.0	12.3
Deck width	(m)	Empirical	20.5	12.9
Girder spacing	(cm)	Empirical	289.6	100.1
Top flange thickness	(cm)	Empirical		
Reinforced concrete			21.3	2.8
Pre-stressed concrete			20.8	2.5
Bottom flange thickness	(cm)	Uniform	11.4	16.5
Wall thickness	(cm)	Uniform	25.4	30.5
Depth of superstructure	(cm)	Uniform		
Reinforced concrete		Uniform	0.055* Span length	0.06* Span length
Pre-stressed concrete		Uniform	0.04* Span length	0.045* Span length
Column diameter	(cm)	Randomly assign 25% of simulation to each 122, 152, 168, 183		
Longitudinal reinforcement ratio	N/A	Uniform		
Pre-1970 design era			1.4	2.4
1970–1990 design era			1.0	3.7
Post-1990 design era			1.0	3.5
Confinement ratio	N/A	Uniform		
Pre-1970 design era			Spacing: 30.5 cm	
1970–1990 design era			0.3	0.9
Post-1990 design era			0.4	1.7
Abutment backwall height	(m)	Uniform	1.1	2.6
Pile spacing	(m)	Uniform	1.7	2.1
Foundation translational stiffness	(kN/cm)	Normal		
Single column - 6 ft dia column 1% long. steel			2977.2	1401.0
Single column - 6 ft dia column 3% long. steel			2451.8	1050.8
Multi-columns - 3 ft dia column 1.5% long. steel			1401.0	1050.8
Foundation rotational stiffness	(kN-m/rad)	Normal		
Single column – 6 ft dia column 1% long. steel			4632.4	1355.8
Single column – 6 ft dia column 3% long. steel			7344.0	1129.8
Multi-columns – 3 ft dia column 1.5% long. steel			0	0
Restrainer length	(m)	Uniform	2.4	6.1
Initial slack in restrainer cable	(mm)	Uniform	6.4	25.4
Restrainer stiffness	(kN/cm)	Uniform	56.9	22.8
Restrainer yield deformation	(mm)	Uniform	38.1	88.9
Number of restrainers	N/A	Uniform	8	50
Concrete compressive strength	(Mpa)	Normal	34.5	4.3
Reinforcing steel yield strength	(Mpa)	Lognormal	6.14	2.0
Shear key capacity	(kN)	Normal	4884.2	646.8
Multiplicative factor for coefficient of friction of bearing pads	N/A	Lognormal	0	0.1
Shear modulus of elastomeric bearing pads	(Mpa)	Uniform	551.6	1723.9
Transverse gap between deck and shear keys	(mm)	Uniform	0	38.1
Longitudinal gap between deck and abutment	(mm)	Uniform	0	152.4
Pile stiffness	(kN/cm)	Lognormal	80.6	0.86
Mass factor	N/A	Uniform	1.1	1.4
Damping	%	Normal	0.045	0.0125

*, ** Factors 1 and 2 represent the mean and standard deviation for normal, lognormal, and empirical distributions; lower bound and upper bound for uniform distribution.

Table 4
Uncertainty lognormal distribution parameters for the column height according to the bridge inventory.

Parameter	Design era	Min	Max	Mean	Standard deviation
Normal column heights	Pre-1971	5.0 (m)	8.6 (m)	1.880	−1.050
(H_{base})	1971–1990	5.0 (m)	10.0 (m)	1.959	−1.016
	Post-1990	5.1 (m)	11.3 (m)	2.031	−0.990
Ratio for tall column heights	Pre-1971	0.56	4.56	0.715	0.267
(H_{ave}/H_{base})	1971–1990	0.65	4.17	0.729	0.237
	Post-1990	0.49	3.97	0.697	0.232
Ratio for unbalanced frames	Pre-1971	0.29	2.33	−0.005	0.237
(H_i/H_{ave})	1971–1990	0.37	1.69	−0.023	0.208
	Post-1990	0.23	2.34	0.0005	0.270

directly affects the number of corners in the Lasso penalty ball, an increase in the problem dimension increases the possibility of more vanishing coefficients. This phenomenon is unlikely to occur in the case of Ridge, simply because of the rounded boundary of the penalty ball.

A critical step in solving a problem using Lasso is finding the optimum value for the tuning parameter λ. In the extreme limit, $\lambda = 0$ reduces the problem to the least square problem, while increasing λ increases the sparsity of the resulting coefficients, until a null model is obtained. In this paper, the typical ten-fold cross-validation is performed to determine the optimum value of λ for which the error calculated by the cross-validation method is smallest [32–34].

3.3. Implementation of statistical methods

In order to perform seismic analysis, 160 sets of ground motions selected by Baker [36] for probabilistic seismic response assess-

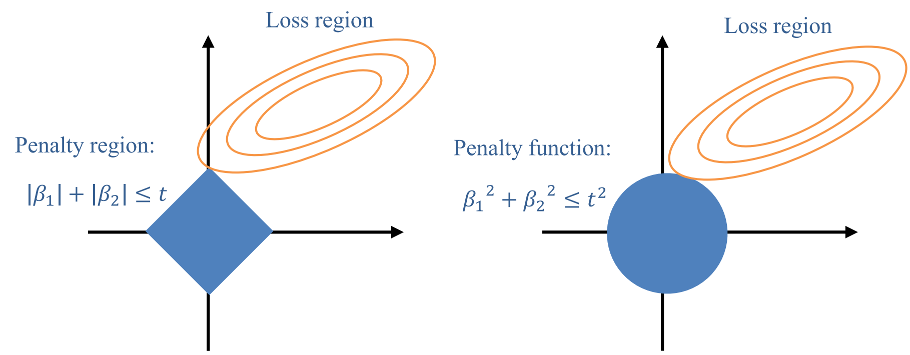

Fig. 4. Illustration of Eqs. (4) and (5) and the difference between the constraints of Lasso and Ridge.

ment of bridges in California are adopted in this study to cover a range of ground motion intensity levels and characteristics. For each of the required finite element models discussed in Section 2, 160 numerical bridge models are generated by sampling across the distribution of the modeling parameters using Latin Hypercube Sampling. Then, the generated bridge models and the ground motions are paired randomly. Nonlinear Time History Analysis (NLTHA) is performed on the bridge models with 11,520 simulations in total, considering twelve irregularity levels with reference to the levels listed in Table 1, two abutment types, and three different design eras (Table 2). The analysis results provide the seismic response of each bridge component, and the peak responses are commonly used to develop the PSDMs. The seismic responses monitored in NLTHA are listed in Table 5, and the impact of variables is evaluated on each of these responses individually. The statistical techniques explained in the previous sections assume a linear relationship between the response and predictors. However, Cornell, et al. [37] proved that the relationship between the bridge seismic response (S_D) and the ground motion intensity (IM) is expressed as a power function $S_D = a(IM)^b$. In logarithmic scale, this regression is converted to a simple linear model $\ln(S_D) = \ln(a) + b\ln(IM)$ where $\ln(a)$ and b are the regression coefficients. Therefore, in this paper, the logarithmic format need to be considered for the relationship between the bridge responses (e.g. column curvature ductility φ) and the potential predictors (e.g. column height) to be able to implement the statistical techniques. In order to extract the effect of various parameters in this study (Table 2), seismic demands ($\ln(Y_i)$) shown in Table 5 are regressed against the ground motion intensity ($X_1 = \ln(IM)$) as well as the remaining predictors ($X_2, \ldots, X_{43}$) that are considered in the transformed logarithmic form. In the following sections, the results of the sensitivity studies are presented to determine the effect of each parameter on the bridge responses.

Table 5
Seismic demand of various components of a bridge.

Component	Engineering Demand Parameter	Units	Assigned Variable
Columns	Curvature (φ)	$(cm)^{-1}$	$Y_1 = \ln(\varphi)$
Deck	Displacement (δ_d)	(cm)	$Y_2 = \ln(\delta_d)$
Foundation rotation	Rotation (θ_f)	(radian)	$Y_3 = \ln(\theta_f)$
Foundation translation	Displacement (δ_f)	(cm)	$Y_4 = \ln(\delta_f)$
Active abutment displacement	Displacement (δ_a)	(cm)	$Y_5 = \ln(\delta_a)$
Passive abutment displacement	Displacement (δ_p)	(cm)	$Y_6 = \ln(\delta_p)$
Transverse abutment displacement	Displacement (δ_t)	(cm)	$Y_7 = \ln(\delta_t)$
Elastomeric bearing pads	Displacement (δ_b)	(cm)	$Y_8 = \ln(\delta_b)$
Shear key	Displacement (δ_s)	(cm)	$Y_9 = \ln(\delta_s)$

4. Results and discussion

Implementing Lasso regression, a sensitivity study was completed to assess the effect of varying the entire list of modeling covariates (Table 2) on the response of the key bridge components including columns, abutments, and bearings (Table 5).

In order to assess the influence of potential predictors, a sensitivity analysis was performed on each irregularity range individually. For example, for the case of unbalanced bridges, the analysis was conducted for the four different ranges of column heights: slightly, moderately, highly, and extremely unbalanced. The results of each range are discussed in Section 4.1. These results provide insight as to which bridge parameters (categorical and numerical) are most important for predicting the seismic response of irregular bridges, and if the set of predictors changes as the *level of irregularity* (e.g. angle of skew) changes. Additionally, to provide an overall perspective of irregular bridge performance, another analysis was performed by considering all different ranges of irregularities in a single group. In this case, the results of the family of tall, unbalanced, and skewed bridges can be more readily compared to identify which predictors are more important for different *types of bridge irregularities*. Corresponding findings are discussed in Section 4.2.

4.1. Analysis of the results for various levels of irregularity

A detailed investigation of the results for each irregularity range is discussed in this section. The findings presented in this section benefit structural engineers in the field of design or those in the field of retrofit by enhancing their understanding of the irregular bridge performance, particularly in each range of irregularity. Selected results are shown in the tables (Tables 6–8) while a comprehensive list of findings is provided in the content of the following subsections. The evaluated responses and the potential predictors are listed in rows and columns, respectively. The shaded cells indicate that the parameter located in that column is identified as an important predictor for estimating the response in the corresponding row. In all cases, the ground motion intensity measure is identified as a certain predictor, as anticipated according to the past studies [37,7].

4.1.1. Tall bridges

As explained in Section 2, three different ranges of tall bridges are investigated in this study. Apparently, the impact of parameters on the responses is varying among ranges of moderately tall, very tall, and extremely tall bridges. An example of results for the extremely tall level is shown in Table 6.

The eliminated parameters from all responses in all three ranges of the tall bridge family are the superstructure box type and the wall thickness. In addition, the restrainer length and the

Table 6
Identified influential parameters for extremely tall bridges.

Extremely Tall	Predictors (X_i)																					
i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Y_1	■			■	■		■	■	■	■	■					■	■	■	■	■	■	■
Y_2	■	■	■	■	■		■	■	■	■	■	■				■	■	■	■	■	■	■
Y_3	■	■		■	■		■	■	■	■	■	■	■	■		■	■	■	■	■	■	■
Y_4	■			■	■		■	■	■	■	■					■	■	■	■	■	■	■
Y_5	■	■		■	■			■	■	■	■						■	■	■	■	■	■
Y_6	■	■		■	■			■	■	■	■						■	■	■	■	■	■
Y_7	■	■	■	■	■		■	■	■	■	■		■			■	■	■	■	■	■	■
Y_8	■	■			■			■		■			■									
Y_9	■		■	■	■		■	■	■	■	■	■				■	■	■		■	■	■

Extremely Tall	Predictors (X_i)																					
i	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	
Y_1	■	■			■		■			■				■	■	■	■		■			
Y_2	■	■				■		■	■	■		■	■	■	■	■	■		■			
Y_3	■	■				■		■	■	■		■	■	■	■	■	■		■			
Y_4	■				■	■		■		■		■		■	■		■		■			
Y_5	■	■					■			■		■		■	■	■						
Y_6	■	■					■			■		■		■	■	■						
Y_7	■	■				■		■	■	■	■	■	■	■	■	■	■		■			
Y_8				■						■					■				■			
Y_9	■		■		■	■	■	■	■	■	■	■	■	■	■	■	■		■			

restrainer yield deformation are among the eliminated list of parameters for the very tall bridges. The least important parameters contributing to only one of the bridge responses are:

- the girder spacing and the restrainer stiffness, for moderately tall bridges;
- the superstructure type, the top flange thickness, and the reinforcing steel yield strength, for very tall bridges;
- the superstructure type, the bottom flange thickness, the restrainer stiffness, and the initial slack in restrainer, for extremely tall bridges.

The most important parameters in predicting the bridge response for three tall ranges are identified to be the mass factor, the normal column height, and the tall ratio. The last two define the bridge column dimension. The following parameters are also identified among the influential parameters for most cases: the abutment backwall height for all responses of moderately tall and very tall bridges, the soil type for extremely tall, and the pile spacing for moderately tall bridges. Several parameters are found to be influential in predicting all responses except bearing displacement. The list includes:

- number of columns per bent, span length, deck width, column diameter, foundation translational and rotational stiffness, for all three ranges;
- abutment backwall height, for extremely tall bridges;
- ground motion time step, for moderately tall and very tall bridges;
- pile stiffness and pile spacing, for very tall and extremely tall bridges.

Aside from the overall bridge response, a number of parameters are highlighted in tables because of their influence on specific responses. As an example, the number of cells affects deck displacement, foundation translation and rotation in all three categories of tall bridges. The direction of applied ground motion is a certain predictor for all responses of extremely tall bridges, however in the case of very tall and moderately tall bridges it is affecting some responses including column curvature, deck displacement, foundation translational and rotational responses, and transverse abutment response. This shows the sensitivity of bridge responses varies with changes in irregularity range. As another example, even though damping ratio affects moderately tall bridge responses, it shows less impact as the tall ratio increases. Likewise, the influence of parameters depends on the regarded bridge responses. For instance, the superstructure depth is recognized as a predictor for column curvature ductility, deck displacement, foundation translation and rotation, and transverse abutment response, while it is not listed in the identified predictors of shear key and bearing displacement. Another example is the shear key capacity that mostly controls the shear key response, transverse abutment displacement, and foundation translation in all irregularity ranges, however, this variable rarely affects the other responses. Apparently, shear key, deck, and foundation translational and rotational displacements rely on the transverse gap between the bridge deck and the shear key.

As previously stated, Lasso picks the most significant potential predictors in the final regression model and eliminates the ones with negligible effects. In some problems, potential correlation appears between the variables, and Lasso keeps the ones with remarkable effects to reduce complexity. Design era demonstrates this concept as it is recognized as an important predictor only in a couple of scenarios: deck displacement of very tall and extremely tall bridges, shear key displacement of all three ranges, and the transverse abutment displacement of extremely tall class. This finding should be interpreted cautiously as it does not simply denote the minimal effect of the design era on the bridge response, yet the results indicate that some other predictors correlated to the design era are previously included in the model. For this problem, longitudinal reinforcement ratio and confinement spacing that are functions of design era are specified as critical parameters particularly in estimating the column curvature, deck displacement, and foundation responses. Similarly, abutment type is determined as a predictor for all responses of extremely tall bridges, while it is a predictor in a couple of responses for very tall and moderately tall bridges, because other parameters like the longitudinal gap between deck and abutment are previously accounted in the model and are sufficient to predict the response.

Overall, speaking of the tall bridge family, a number of geometric specifications such as the column height and tall ratio are identified as the most influential predictors for the class of tall bridges. Among the material properties, concrete compressive strength shows the highest effectiveness on the bridge response. The influence of some parameters shows similar trends for various irregularity ranges, and they could eventually be classified as the most or least influential predictors, although the effect of some other parameters depends on particular responses or unique irregularity range.

4.1.2. Unbalanced bridges

As mentioned earlier, four different ranges of unbalanced bridges are evaluated in this study. An example of the results for the extremely unbalanced level is shown in Table 7. The important observed trends are explained herein and more details are provided in the tables.

In the unbalanced bridge family, the most significant parameters identified for almost all bridge responses are listed as the longitudinal reinforcement ratio, direction of applied ground motion, pile stiffness, and geometric attributes including span length, column height along with the short and tall ratios, deck width and column diameter. These geometric features are found to be less effective on shear key and bearing displacement. The bearing response in highly and extremely unbalanced range does not show sensitivity to the pile stiffness. In addition, foundation translational stiffness is highlighted in most of the responses excluding foundation rotation of moderately and extremely unbalanced bridges and shear key displacement of ranges 2–4 of unbalanced class.

The bridge responses show more sensitivity to a number of parameters as the unbalanced irregularity range moves from the slight to the extreme. The abutment backwall height, soil type, pile spacing and stiffness, and concrete compressive strength are examples of these parameters. Moreover, the column height, particularly the shorter column height, shows impact on more responses as the irregularity range increases. Additional examples are the transverse gap between deck and shear key, and the longitudinal gap between deck and abutment with no effect on the column curvature, deck displacement, and foundation rotation in the slightly unbalanced range. Unlike that scenario, as the irregularity range increases to the extreme, the bridge response displays less sensitivity to a few of the parameters including superstructure type and girder spacing.

Parameters with the least impact on the responses include wall and flange thickness. The superstructure depth affects the abutment responses in all ranges of unbalanced irregularity, however it influences deck, foundation, and shear key responses of slightly, moderately, and highly unbalanced bridges. Abutment backwall height is an important predictor to capture the abutment response as well as the bearing response in all cases. In addition, the backwall height is highlighted as a predictor of deck displacement for moderately and extremely unbalanced bridges, for foundation translational displacement of slightly and extremely unbalanced range, and for the column curvature ductility of extremely unbalanced class. In the range of moderately, highly, and extremely unbalanced bridges, shear key capacity only influences shear key displacement, while it affects abutment response of slightly unbalanced bridges.

4.1.3. Skewed bridges

As noted in Section 2, five ranges of skew angles are considered in this study. An example of the results for the extremely skewed level is shown in Table 8. Concentrating on the skewed bridges, wall thickness, superstructure depth, and bottom flange thickness are determined as the least important predictors for most of the responses. On the other hand, span length, longitudinal reinforcement ratio, and pile stiffness are diagnosed to be imperative for all bridge responses in the entire range of skewed bridges. The

Table 7
Identified influential parameters for extremely unbalanced bridges.

	Predictors (X_i)																					
i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Extremely Unbalanced Y_1	■	■						■	■	■	■					■	■	■		■	■	■
Y_2	■	■						■	■	■	■					■	■	■		■	■	■
Y_3	■		■	■	■		■	■	■	■	■	■	■			■	■	■		■	■	■
Y_4	■	■		■	■		■	■	■	■	■	■	■				■	■	■	■	■	■
Y_5	■			■	■		■	■	■	■	■	■				■	■	■	■	■	■	■
Y_6	■			■	■		■	■	■	■	■					■	■	■	■	■		■
Y_7	■				■			■	■	■	■	■		■		■	■	■		■	■	■
Y_8	■				■	■		■		■						■				■	■	■
Y_9	■				■	■		■								■		■				

	Predictors (X_i)																					
i	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	
Extremely Unbalanced Y_1	■			■				■				■	■	■	■	■				■	■	
Y_2	■			■								■	■	■	■					■	■	
Y_3	■		■	■		■		■		■		■	■	■	■	■	■			■	■	
Y_4	■			■		■	■			■		■	■	■	■	■	■			■		
Y_5	■		■		■	■		■			■	■	■	■	■	■	■			■	■	
Y_6	■				■			■		■	■	■	■	■	■		■			■		
Y_7			■		■	■		■			■	■	■	■	■	■	■			■		
Y_8						■	■			■	■	■	■		■					■	■	
Y_9	■				■	■	■		■	■		■					■			■		

Table 8
Identified influential parameters for extremely skewed bridges.

Skewed (Extreme) — Predictors (X_i)

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Y_1	■		■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■	■
Y_2	■		■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■	■
Y_3	■		■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■	■
Y_4	■		■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■	■
Y_5	■			■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■	■
Y_6	■		■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■	■
Y_7	■		■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■	■
Y_8	■		■					■	■	■			■			■	■	■		■	■	
Y_9	■		■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■	■

Skewed (Extreme) — Predictors (X_i)

i	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
Y_1	■	■	■	■		■	■	■	■	■	■	■	■	■	■	■	■	■			
Y_2	■	■	■	■		■	■	■	■	■	■	■	■	■	■	■	■	■			
Y_3	■		■	■	■	■	■	■	■	■	■	■		■	■	■	■	■			
Y_4	■		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■			
Y_5	■				■	■		■	■	■		■	■	■	■	■	■	■			
Y_6	■		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■			
Y_7	■		■	■	■	■		■	■	■		■	■	■	■	■	■	■			
Y_8					■	■		■		■		■	■	■				■			
Y_9		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■			

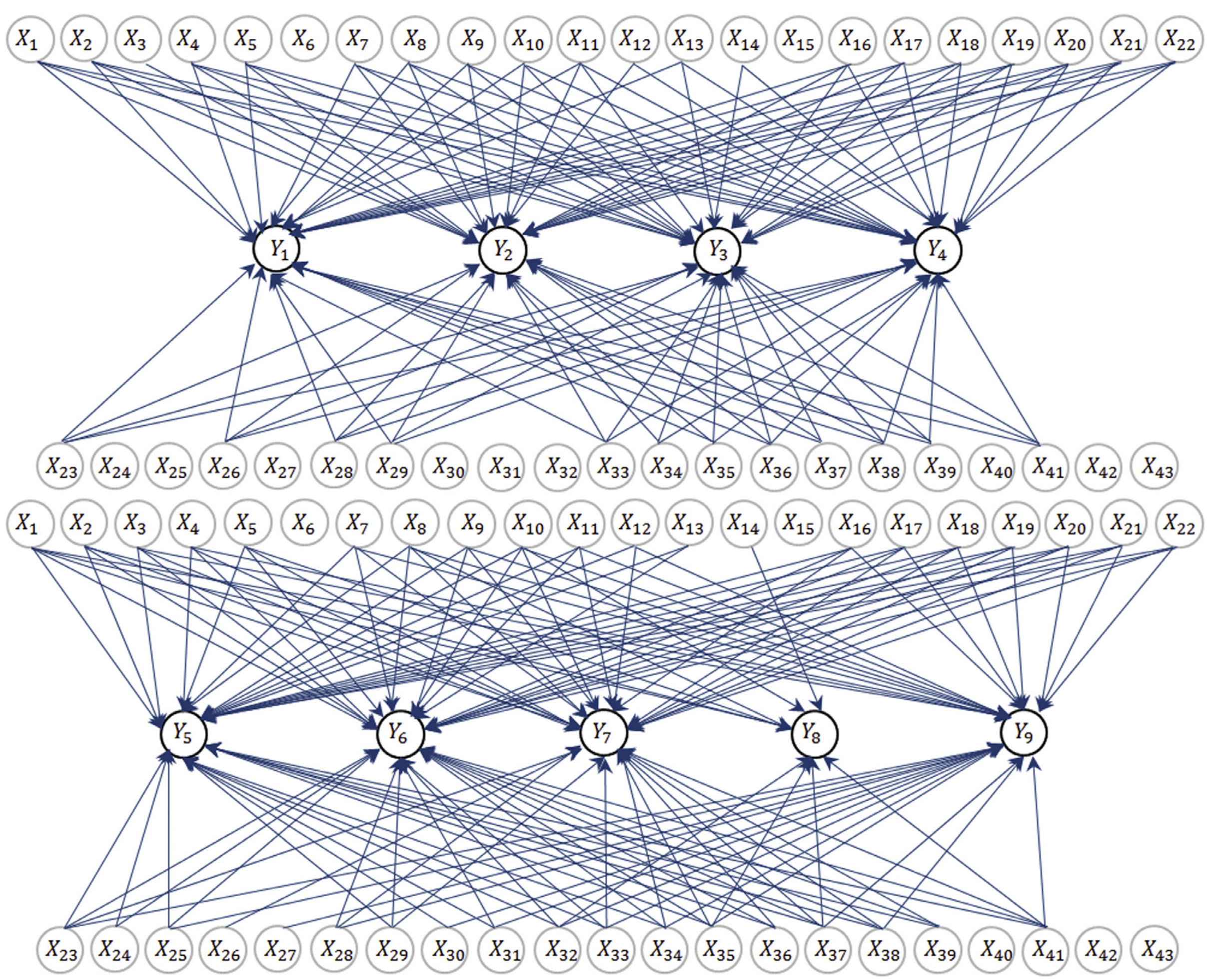

Fig. 5. System network diagram for the class of tall bridges (Appendix B).

column height is highlighted in most of the responses and skew angles. According to the analysis results, multiple parameters influence most of the bridge responses, however they indicate less weight in estimating the shear key and bearing responses in some ranges of skew angles. These variables include soil type, confinement spacing, mass factor, deck width, foundation translational and rotational stiffness, and top flange thickness. Another group of parameters consists of the ones with effects on the whole responses except the shear key. Examples of these are abutment backwall height and pile spacing for the overall skewed bridges, and column diameter which displays less impact on shear key for bridges with low to very high skew angles.

A few other significant bridge attributes for skewed family are longitudinal gap between deck and abutment, transverse gap between deck and shear key, and skew angle. The longitudinal gap affects most of the skewed bridge responses with a couple of exceptions such as shear key displacement of bridges with low to medium skew angles, and foundation rotation and transverse abutment displacement in a few skew ranges. The transverse gap influences all responses of bridges with low and extreme skew angles, while it is mostly critical for the abutment, shear key, and bearing response of moderate, high, and very high classes.

The analysis results detected skew angle to be most effective in predicting the majority of bridge responses listed as column curvature, deck displacement, foundation rotation, abutment active and passive displacements, and bearing and shear key responses, even though not to be significant in predicting the shear key displacement of the low to medium skew levels. Compared to the aforementioned responses, the skew angle is noted less effective on controlling the foundation translation and transverse abutment displacement of moderately, highly, and very highly skewed bridges.

Similar to tall and unbalanced bridges, the impact of parameters varies within various levels of skew angles. For instance, the superstructure type and number of box cells present impact reduction as the skew angle increases from low to very high, despite the fact that they are important predictors for extremely skewed bridges. Overall, the sensitivity of almost all bridge responses is enhanced for the extremely skewed bridges compared to the other ranges.

4.2. Analysis of the results for different types of bridge irregularity

The previous section explained the effects of input parameters on the seismic response for each irregularity range of interest. Although the comparisons provide a comprehensive assessment of each individual range, it makes driving a general conclusion intricate. This section provides the analysis for the whole family of tall (Fig. 5 in this section), unbalanced (Fig. 6 in Appendix A), and skewed bridges (Fig. 7 in Appendix A) by considering all ranges in a single group instead of focusing the analysis on individual

Table 9
Comparative analysis of the identified influential parameters.

Tall Bridge Class		Unbalanced Bridge Class		Skewed Bridge Class	
Partial F-test result	Regressors X_i	Partial F-test result	Regressors X_i	Partial F-test result	Regressors X_i
F	i	F	i	F	i
841.34	1	6347.1	1	2577.7	1
140.81	41	370.94	18	178.82	18
109.32	10	109.82	9	61.288	17
96.924	18	37.025	42	49.072	28
40.857	9	25.951	29	31.284	37
23.062	21	22.980	43	18.469	13
19.209	17	21.492	13	12.650	12
16.681	29	18.589	37	11.892	4
15.644	37	16.517	8	11.210	10
14.857	13	15.786	7	10.788	21
13.240	33	10.771	4	9.4173	3
12.455	4	6.1803	22	7.9154	36
12.317	28	2.9170	39	6.8257	31
12.254	39	2.6458	38	6.6300	40
9.8648	7	2.5959	6	6.0540	19
5.1979	2	2.1159	34	4.8201	11
5.0957	19	1.2465	19	3.6175	6
3.8813	8	1.0973	21	3.6167	16
2.3597	16	0.6394	35	3.6146	9
2.0457	38	0.3347	20	2.6915	23
1.3606	20	0.3155	5	2.0644	8
1.2597	36	0.2522	10	1.6892	33
1.1826	5	0.0342	17	1.3645	32
1.1703	23	0.0109	23	1.2809	20
0.8765	22	0.0093	2	1.1881	29
0.1750	35	0.0086	24	0.9322	7
0.0094	11	0.0085	27	0.8733	25
0.0086	26			0.8681	27
				0.8649	2
				0.8611	26
				0.8574	5
				0.5333	30
				0.4928	35
				0.3908	22
				0.1407	39
				0.0378	34
				0.0108	38

ranges. This makes the results of the three different irregularity types comparable.

The results confirm the significant influence of the ground motion intensity in all cases. The longitudinal gap between deck and abutment, a function of abutment type, is an influential predictor in the total list. However, the variable corresponding to the abutment type displays more importance on tall classes since it contributes to every one of the responses. Deck width sounds less important in the unbalanced class, while it shows essence in most of the responses in the tall and skewed categories. Although superstructure type has no impact on any of the responses of tall bridges, it has an effect in most cases of the other two irregularity types. Wall thickness has no impact on any of the responses. However, several parameters are detected as influential predictors for the majority of scenarios regardless of the irregularity type. Such variables are the column's height and diameter, number of columns per bent, span length, abutment backwall height, pile's stiffness and spacing, foundation translational stiffness, soil type, transverse gap between deck and shear key, mass factor, damping ratio, and ground motion characteristics.

Evidently, more predictors are involved in the final response model of skewed bridges compared to those of unbalanced and tall, indicating higher sensitivity of skewed bridges to the modeling parameters. Additionally, a number of parameters are detected to be more effective for skewed bridges than the other two irregularity types. Design era is an example of this, even though the reinforcement detailing that is a function of design era is imperative in most scenarios irrespective of the irregularity class. Additional parameters include girder spacing, foundation rotational stiffness, material properties, and shear key capacity. The mentioned observation relates to the distinctive seismic behavior of skewed bridges and their tendency to experience larger deck displacements and unseating between the deck and the abutment.

4.3. Comparative analysis of influential parameters

Although the applied regression analysis picked the influential predictors among the entire list of possible covariates, the interpretation is challenging to rank the final predictors. A reliable and well-known technique to compare the importance of a predictor with respect to the other ones is the partial F-test statistic [35]. In this study, the partial F-test is applied to test the hypothesis that q coefficients are zero while the full model includes l coefficients. This method compares the residual sum of squares (RSS) for two separate regression models: one is the full model representing the final Lasso regression model and the other one is the reduced model accounting for the full model eliminating variable of interest X_i. This method provides

$$F = \frac{\frac{RSS_{reduced} - RSS_{full}}{q}}{\frac{RSS_{full}}{m-l}}, \tag{6}$$

representing the ratio of two variances. The denominator equals to the mean squared error of the full model divided by its degrees of freedom $m - l$. The numerator computes the difference in the RSS produced by the q variables, divided by the number of eliminated variables. Table 9 provides a comparative analysis of the influential parameters on the bridge column response. Higher F-values are equivalent to the lower probability of rejecting the hypothesis that q coefficients are zero. Consequently, the variables with higher F-values have a higher impact on the estimated response Y_i. In this study, the test was performed on individual variables ($q = 1$) to compare their relative importance.

The contribution of each parameter to the seismic response can be evaluated using Table 9. Apparently, ground motion intensity with the highest F-value is recognized as the most influential parameter in all three irregularity types. For the other parameters, the horizontal bars represent the single parameter contribution in estimating the response.

According to the results, the column height and tall ratio are most heavily involved in the predicted response of tall bridges since both control the column's overall strength. The third significant feature is the longitudinal reinforcement ratio dominating the column's ductility and seismic performance. The next variable with the highest weight is the span length because of its contribution to the force and dead load applied to the columns. Subsequent components are pile spacing, column diameter, concrete compressive strength, number of columns per bent, and mass factor, mainly associated with the column's seismic resistance. Moreover, one of the main observed damage states relates to the cracking and breaking of tall piers, in addition to the buckling of their steel reinforcements. Consequently, stronger column bents are required to be designed for tall bridges to resist large bending moments, shear forces, and torques.

Regarding the unbalanced frame bridges, the comparative analysis results assign the highest impact to the longitudinal reinforcement ratio, span length, short and tall column height ratios, concrete compressive strength, number of columns per bent, and foundation translational stiffness. Similar to the tall category, many of the parameters primarily define the column's strength and ductility which eventually impacts the general seismic performance of the bridge, as columns are known as the most vulnerable bridge components. As stated in the Introduction, according to the post-earthquake investigations, unequal force distribution within the columns of an unbalanced bridge causes the column's damage leading to the superstructure's failure. A number of noted significant parameters including the span length and the column height ratios control the force distribution among various columns within a bridge frame. As a result, particular attention in designing shorter columns, particularly the ones adjacent to the very tall columns, is necessitated to increase their seismic resistance and enhance the overall performance of the bridge.

Compared to the tall and unbalanced irregularity, skew increases the sensitivity of bridge responses to the uncertain parameters. The results imply higher importance of the longitudinal reinforcement ratios, column diameter, number of restrainers, mass factor, number of columns per bent, column height, pile spacing and stiffness, design era, shear key capacity, skew angle, confinement spacing and deck width, versus the remaining significant predictors. The post-earthquake investigation of skewed bridges showed serious damage caused by displacement or unseating of the bridge deck and larger demands including column forces. Apparently, every one of the outlined parameters significantly contributes in both demands. The column specifications such as column diameter and height as well as the reinforcement detailing provide the general column's durability. Besides, span length, deck width, pile properties, skew angle, and shear key capacity contribute more on the other demands such as the deck displacements that indirectly have an effect on the column's demand.

The significance of individual influential parameters contributing to each class of irregularity is compared above in Table 9. The statistical analysis reveals that the commonly detected parameters including ground motion intensity, longitudinal reinforcement ratio, column diameter, number of columns per bent, column height, span length, and concrete compressive strength significantly influence the response of all three studied irregular bridges. Amongst these, longitudinal reinforcement ratio dominates the column response, as the horizontal bars in Table 9 show.

5. Conclusions

In order to improve our understanding of the seismic performance of irregular bridges, this study examines the influential parameters of irregular bridges by performing sensitivity analyses of skewed, tall columns, and unbalanced bridge frames. This study is beneficial in enhancing the knowledge regarding the effect of a broad range of parameters, associated with the finite element bridge models, on the seismic response of bridges with geometric irregularities in their configuration.

Moreover, in contrast to the previous sensitivity studies, categorical regression analysis is conducted to uncover the influence of variables affecting the seismic responses. This is advantageous since a number of parameters that describe irregular bridges are categorical in nature or can be easily classified as such in order to simplify database development regarding bridge characteristics that inform vulnerability modeling. A robust and reliable statistical tool, Lasso Regression, is implemented in this paper providing the opportunity to involve both categorical and numerical covariates in the regression model. The presented results reveal that the influence of a parameter varies with increasing irregularity ranges. For instance, bridges with skew angles beyond the medium level and particularly beyond 60° display higher sensitivity to the modeling parameters. Similar trends are observed for tall and unbalanced categories.

This paper implements statistical techniques to explicitly identify the most and the least influential parameters on the bridge responses related to the column, deck, foundation, abutment, bearing, and shear key. Although the relative significance of the various predictors changes over different bridge responses and irregularity ranges, there exist many common influential parameters including ground motion intensity, longitudinal reinforcement ratio, column diameter, number of columns per bent, column height, span length, and concrete compressive strength.

The influential parameters identified in this paper should to be included in the seismic demand modeling of irregular bridges. In order to accomplish this, bridge databases are required to be enhanced to provide realistic information for the influential parameters. Where the information is not accessible, suitable uncertainty in the assignment of these parameters need to be considered in the modeling of bridge performance. It is found that irregularity parameters play essential roles in the seismic response estimations for almost all scenarios. Although the statistical approach is applied herein for identifying significant parameters affecting concrete box-bridges, the similar methodology can be applied to other bridge types such as steel-girder, T-girder, and slab bridges. Since the current study was performed based on representative bridge models with existing bridge characteristics, the results are advantageous in informing regional seismic risk and fragility assessment of irregular bridges.

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Appendix A

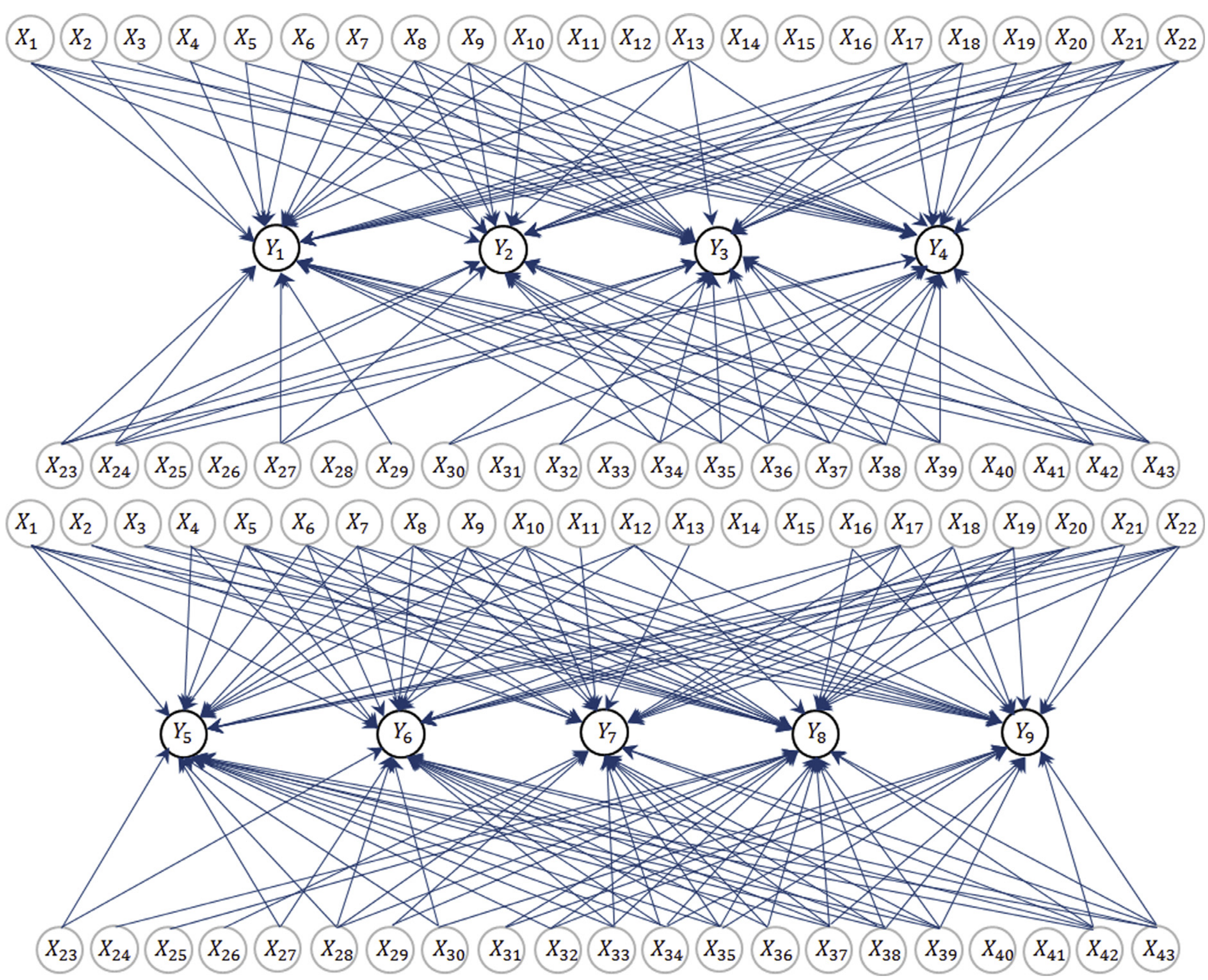

Fig. 6. System network diagram for the class of unbalanced bridges.

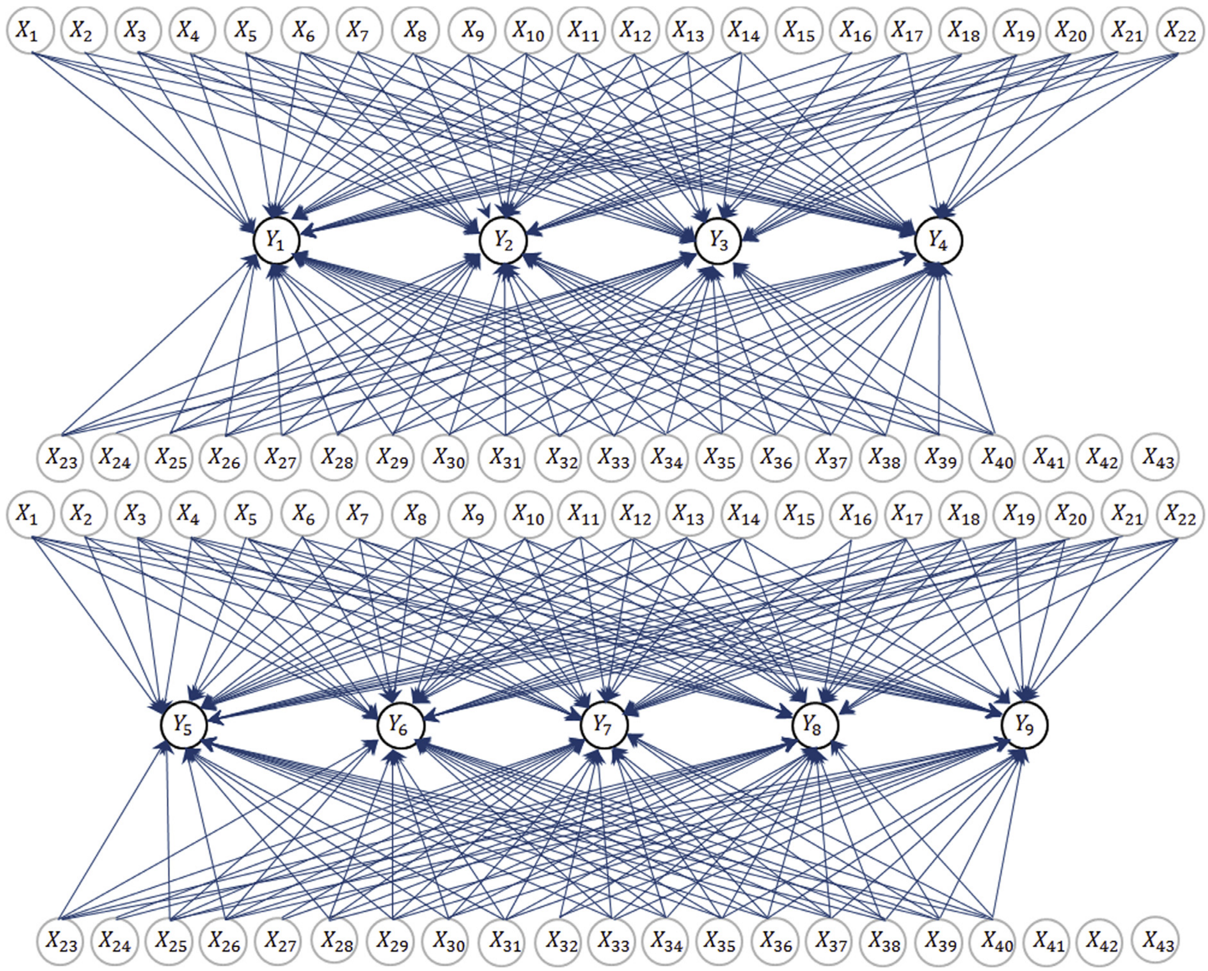

Fig. 7. System network diagram for the class of skewed bridges.

Table 10
Values for the coefficients in Eq. (7).

Coefficients	Values	Coefficients	Values	Coefficients	Values	Coefficients	Values
Intercept (β_0)	11.673	$\beta_{7_{nine-cells}}$	−0.199	β_{18}	−1.270	β_{33}	−0.619
β_1	1.133	$\beta_{7_{eleven-cells}}$	−0.278	β_{19}	−0.279	β_{35}	−0.020
$\beta_{2_{seat}}$	1.988	$\beta_{8_{trans}}$	0.142	β_{20}	0.167	β_{36}	−0.505
$\beta_{4_{two-col}}$	−0.942	β_9	2.185	β_{21}	−5.288	β_{37}	2.205
$\beta_{4_{three-col}}$	4.638	β_{10}	−2.012	β_{22}	0.058	β_{38}	0.162
$\beta_{4_{four-col}}$	3.518	β_{11}	−0.073	β_{23}	0.223	β_{39}	−0.275
$\beta_{4_{five-col}}$	2.81	β_{13}	8.382	β_{26}	−0.021	β_{41}	−1.421
$\beta_{5_{sand}}$	0.078	β_{16}	0.419	β_{28}	−0.607		
$\beta_{7_{seven-cells}}$	−1.582	β_{17}	−4.425	β_{29}	−1.116		

Appendix B

An example of the statistical response surface model is presented for the column's response of the tall bridge class as

$$\ln(\hat{Y}_{1_{tall}}) = \beta_0 + \beta_1 \ln(X_1) + \sum_{j=2}^{8} \boldsymbol{\beta}_j \boldsymbol{X}_j + \sum_{j=9}^{43} \beta_j \ln(X_j), \tag{7}$$

where the coefficients are listed in Table 10. In Eq. (7), $\boldsymbol{\beta}_j$ and $\boldsymbol{X}_j$ ($j = 2, \ldots, 8$) are shown as vectors since they include dummy variables for each categorical variable.

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